

**Core Knowledge
Mathematics for Teachers I:
Basic Number Concepts
Syllabus**

This is a syllabus for the first half of a two-semester course in mathematics for prospective K-8 teachers. It is intended to prepare teachers to teach in Core Knowledge Schools and other schools with challenging mathematics standards. Because Mathematics for Teachers II builds on knowledge acquired in this course and uses the same textbooks, these two courses should be offered in consecutive terms or semesters and this one should be a prerequisite for the sequel.

The syllabus assumes that 13 full weeks are available and provides a breakdown of topics by week. This scheduling of topics is necessarily approximate since different classes of students proceed at different paces, sometimes more quickly one week and more slowly during another.

The syllabus is arranged so that instructors may use one of two widely used textbooks for these courses (but not both). They are:

A Problem Solving Approach to Mathematics for Elementary School Teachers (Seventh Edition) by Billstein, Libeskind, Lott, published by Addison Wesley Longman.

Mathematics for elementary School Teachers: A Contemporary Approach (Fifth Edition), by Musser, Burger, and Peterson, published by Wiley & Sons, Inc.

In addition, this syllabus relies heavily on *The Winning Equation* for classroom presentations. *The Winning Equation* was created by the Curriculum and Instruction Steering Committee (CISC) of the California County Superintendents Educational Services Association (CCSESA). This material consists of master copies of overhead slides, and facilitator's notes, and is part of a California State approved training for grade four through seven teachers, under California's Assembly Bill 1331. *The Winning Equation* modules require an overhead projector, but otherwise they are easy to use. The slides are generally self-explanatory and develop their respective topics coherently and gradually. Instructors need only project them in order, comment on them, and use them to stimulate classroom discussion. The facilitator notes are generally not needed, but they are available to assist instructors. Include time for worksheets, with labels like H-2, H-20, etc. "H" stands for "Handout" and "T" stands for "Transparency." It is not necessary to use the pre- or post-tests from this series, but they are available if the instructor wishes to use them or adapt them. The relevant portions of *The Winning Equation* for this semester are available on Professor David Klein's website (<http://www.csun.edu/~vcmt00m/WE.html>).

The second semester course also relies on the following reference available in popular bookstores:

Algebra Success in 20 Minutes A Day, by Barbara Jund from Learning Express Skill Builders, copyright 2000 LearningExpress, LLC.

It is not essential that the instructor use this particular reference, and this one does have shortcomings, though the exercises are useful. Any material that provides students with practice and review of middle and high school algebra would suffice.

Finally, the syllabus makes reference to the following secondary materials, which may be used at the discretion of the instructor:

Knowing and Teaching Elementary Mathematics, by Liping Ma, published by Lawrence Erlbaum Associates, copyright 1999. This reference can be used for supplementary reading assignments made approximately a week in advance of associated material from the lectures. The syllabus includes suggestions for the use of this book.

Algebra, by I. M. Gelfand and A. Shen, published by Birkhauser, copyright 1993

The Role of Long Division in the K-12 Curriculum, by David Klein and R. James Milgram is posted on the "Mathematically Correct Website" or at: <http://www.mathematicallycorrect.com/> or the paper may be downloaded from: <ftp://math.stanford.edu/pub/papers/milgram/long-division/longdivisiondone.htm>

General Instruction: Calculators should not be used by students in the instructional components for this course, unless explicitly indicated in the syllabus.

Use of this Syllabus: This syllabus was developed by David Klein, professor of mathematics at California State University, Northridge, as part of *What Elementary Teachers Need to Know*, a teacher education initiative developed by the Core Knowledge Foundation. Although the syllabus is copyrighted by the foundation, and may not be marketed by third parties, anyone who wishes to use, reproduce, or adapt it for educational purposes is welcome to do so. However, we do ask individuals using this syllabus to notify us so we can assess the distribution and spread of the syllabi and serve as a repository of information about how they may be improved and more effectively used. Please contact us at <http://coreknowledge.org/CK/contact.htm>.

First Semester

Week 1

Optional for the first day: Give an arithmetic test to make sure that students have the prerequisites for this course. They should be able to carry out whole number computations, decimal computations, calculations involving percents, and fraction calculations using the four arithmetic operations of addition, subtraction, multiplication, and division. Many elementary school teachers and prospective elementary school teachers are unable to carry out arithmetic calculations at the sixth grade level. If a substantial proportion of the students in this course have serious deficits in arithmetic, this will slow down the course and make it difficult to complete all of the material. Such students should be redirected to remedial courses *that do not use calculators*. This course focuses primarily on the reasons behind arithmetic procedures and connections to and basic ideas of algebra, geometry, probability, and statistics. The focus is *not* on how to calculate.

Topic 1. Explain the concepts of set, subset, the empty set, unions, intersections, and complements relative to a given universal set. This topic will be used later for probability. The introduction to addition of whole numbers also depends on concept of sets to some extent. The material in the sequel does not depend on Cartesian products so this topic is optional.

References:

Billstein et al, Section 2.1, "Describing Sets," and Section 2.2, "Other Set Operations and Their Properties"

Musser et al, Section 2.1, "Sets as a Basis for Whole Numbers," Include representative problems for homework to solidify this concept.

Problems for homework should be assigned at the discretion of the instructor from one of these two references

Topic 2. Roman Numerals and historical enumeration systems other than the standard base 10 system. Particular focus should be given to Roman Numerals, as the Core-Knowledge mathematics sequence explicitly includes them, and they continue to be used in modern times. The Core-Knowledge curriculum introduces Babylonian, Egyptian, Greek, and Mayan history in the elementary grades, and it is worth pointing this out to prospective teachers. The numeration systems of these civilizations are explained in the references below. However, the primary emphasis should be on Roman numerals, as they have the greatest practical application of these numeration systems. It is not

necessary for prospective teachers to master the other numeration systems, but they should be aware of the information in the references below.

References:

Billstein et al, Section 3.1, "Numeration Systems"

Musser et al, Section 2.2, "Whole Numbers and Numeration"

Problems for homework should be assigned at the discretion of the instructor from one of these two references

Week 2

Topic 3. Exponents.

Introducing exponents at this stage of the course is unconventional. However, a review of exponential notation together with the basic laws of exponents helps to facilitate the development of number bases, including the standard base 10 number system.

Use the overhead transparency masters to make overhead slides from Primary Content Module 1, (Exponents/ Powers and Roots), of "The Winning Equation." Be sure to emphasize that students may use the results of laws of exponents once they have been established, and they do not have to rederive these laws each time they want to use them. For example, after it has been established that $a^m a^n = a^{m+n}$, this equation can be applied directly to simplify expressions like $5^4 5^{-3}$. This presentation requires 1.5 to 2 hours.

Assign homework problems on exponents from:

Billstein et al, Section 5.3, "Multiplication and Division of Rational Numbers," subsection: "Extending the Notion of Exponents" (use problems 27 through 30 at the end of this section)

or

Musser et al, Section 3.3, "Ordering and Exponents"

Suggested Quiz 1: At the end of the week, give a 20 minute quiz on sets, other enumeration systems (including Roman Numerals), and exponents.

Supplemental Reading Assignment for the next topic: Chapter 1 of "Knowing and Teaching Elementary Mathematics," by Liping Ma.

Week 3

Topic 4. Whole Numbers, Integers, Addition and Subtraction.

Use the overhead transparency masters to make overhead slides from Module 2, (Whole Numbers and Integers) from "The Winning Equation." This presentation requires approximately two hours. The emphasis here is on the commutative and associative properties, and the connection between arithmetic and algebra.

Reading and Problem Assignments:

Billstein et al: Section 2.3, "Addition and Subtraction of Whole Numbers," Section 3.2, "Algorithms for Whole-Number Addition and Subtraction" (but the subsection, "Addition and Subtraction in Bases Other than Ten" is optional and may be skipped), Section 4.1, "Integers and the Operations of Addition and Subtraction." Assign these sections for reading and choose problems from the section exercises for practice on addition and subtraction of integers.

Musser et al: Section 3.1 "Addition and Subtraction," and Section 8.1 "Addition and Subtraction." Assign these sections for reading and choose problems from these sections for practice on addition and subtraction of integers.

Supplemental Reading Assignment for the next topic: Chapter 2 of "Knowing and Teaching Elementary Mathematics," by Liping Ma.

Weeks 4 and 5

Topic 5. Multiplication and Division of Integers.

Use the overhead transparency masters to make overhead slides from Module 3 (Multiplication and Division) from "The Winning Equation." This presentation requires approximately two hours.

Reading Assignments:

Billstein et al: Section 2.4, "Multiplication and Division of Whole Numbers," Section 3.3, "Algorithms for Whole-Number Multiplication and Division," Section 3.4, "Mental Mathematics and Estimation for Whole-Number Operations," and Section 4.2, "Multiplication and Division of Integers." Assign problems from these sections.

Musser et al: Section 3.2 "Multiplication and Division", Section 4.1, "Mental Math, Estimation and Calculators," Section 4.2, "Written Algorithms for Whole Number Operations," and Section 8.2, "Multiplication, Division, and Order." Assign problems from these sections, but avoid the use of calculators especially in Section 4.1. The treatment of scientific notation may be postponed to weeks 10 and 11 (see below).

Weeks 6 and 7

Topic 6. Prime Numbers and Factorization.

Use the overhead transparency masters to make overhead slides from Module 4, (Factors of Whole Numbers) from "The Winning Equation." This presentation requires at least two hours.

Reading Assignment:

Billstein et al: Sections 4.3, "Divisibility," Section 4.4, "Prime and Composite Numbers," and Section 4.5, "Greatest Common Divisor and Least Common Multiple." Assign problems from these sections. Assign or explain problem 34 of Section 4.4 (page 218). This problem is essentially a proof that there are infinitely many prime numbers. Explain the Euclidean algorithm for finding the Greatest Common Divisor of two whole numbers.

Musser et al: Section 5.1 "Primes, Composites, and Test for Divisibility" and Section 5.2, "Counting Factors, Greatest Common Factor, and Least Common Multiple." Explain the Theorem in Section 5.2 that there are infinitely many prime numbers. Assign problems from both sections. Explain the Euclidean algorithm for finding the Greatest Common Divisor of two whole numbers.

Suggested Midterm on Topics through week 5.

Supplemental Reading Assignment for the next topic: Chapter 3 of "Knowing and Teaching Elementary Mathematics," by Liping Ma.

Weeks 8 and 9

Topic 7. Rational Numbers.

Use the overhead transparency masters to make overhead slides from Module 5, (Rational Numbers) from "The Winning Equation." This presentation may require four hours or more. The paper folding projects are optional, and should be skipped if there is a time crunch.

Reading Assignment:

Billstein et al: Chapter 5, "Rational Numbers as Fractions." Assign problems from this chapter.

Musser et al: Chapter 6, "Fractions." Assign problems from this chapter.

Supplemental Assignment: Ask students to invent and write down three word problems each for multiplying and for dividing fractions. Refer students to Chapter 3 of Liping Ma's book (already assigned for reading) for examples.

Weeks 10 and 11

Topic 8. Decimals and Real numbers.

Use the overhead transparency masters to make overhead slides from Module 6, (Fractions and Decimals) from "The Winning Equation." This presentation may require four hours or more.

Reading Assignment:

Billstein et al: Chapter 6, "Decimals, Percents, and Real Numbers." Include the discussion on scientific notation at the end of Section 6.1. Ask students why multiplying a number by 10 moves the decimal point one place value to the right. Be sure students understand this in terms of the distributive property and the base 10 structure of numbers. Assign problems from these sections.

Musser et al: Chapter 7, "Decimals, Ratio, Proportion, and Percent," and Section 9.1, "The Rational Numbers," and Section 9.2, "The Real Numbers." Also, briefly discuss scientific notation, which is treated in Section 8.2 on pages 326 and 327. Ask students why multiplying a number by 10 moves the decimal point one place value to the right. Be sure students understand this in terms of the distributive property and the base 10 structure of numbers. Assign problems from these sections.

Supplementary, optional reference: *The Role of Long Division in the K-12 Curriculum*, by David Klein and R. James Milgram

Enrichment topics: Derive formulas for Simple and Compound interest.

Prove that the square root of 2 is irrational.

Weeks 12 and 13

Topic 9. Probability.

Review material on sets from earlier in the semester, as needed.

Present material from

Billstein et al: Chapter 7, Probability

Musser et al: Chapter 11 and assign that chapter for students to read. Assign problems from that chapter.